Division of something into two equal or congruent parts Not to be confused with Dissection. For the bisection theorem in measure theory, see Ham sandwich theorem. For the root-finding method, see Bisection method. For other uses, see Bisect (disambiguation). Line DE bisects line AB at D, line EF is a perpendicular bisector of segment AD at C, and line EF is the interior bisector of right angle AED. In geometry, bisection is the division of something into two equal or congruent parts (having the same shape and size). Usually it involves a bisecting line, also called a bisector. The most often considered types of bisectors are the segment bisector, a line that passes through the midpoint of a given segment, and the angle bisector, a line that passes through the apex of an angle (that divides it into two equal angles). In three-dimensional space, bisection is usually done by a bisecting plane, also called the bisector. Perpendicular bisector of a line segment The perpendicular bisector of a line segment is a line which meets the segment at its midpoint perpendicularly. The perpendicular bisector of a line segment A B {\displaystyle AB} also has the property that each of its points X {\displaystyle X} is equidistant from segment AB's endpoints: (D) | X A | = | X B | {\displaystyle \quad |XA|=|XB|} . The proof follows from | M A | = | M B | {\displaystyle |MA|=|MB|} and Pythagoras' theorem: | X A | 2 = | X M | 2 + | M A | 2 = | X M | 2 + | M B | 2 = | X B | 2 . {\displaystyle |XA|^{2}=|XM|^{2}+|MA|^{2}=|XM|^{2}+|MB|^{2}=|XB|^{2}\;.} Property (D) is usually used for the construction of a perpendicular bisector: Construction by straight edge and compass In classical geometry, the bisection is a simple compass and straightedge construction, whose possibility depends on the ability to draw arcs of equal radii and different centers: The segment A B {\displaystyle AB} is bisected by drawing intersecting circles of equal radius r > 1 2 | A B | {\displaystyle r>{\tfrac {1}{2}}|AB|} , whose centers are the endpoints of the segment. The line determined by the points of intersection of the two circles is the perpendicular bisector of the segment. Because the construction of the bisector is done without the knowledge of the segment's midpoint M {\displaystyle M} , the construction is used for determining M {\displaystyle M} as the intersection of the bisector and the line segment. This construction is in fact used when constructing a line perpendicular to a given line g {\displaystyle g} at a given point P {\displaystyle P} : drawing a circle whose center is P {\displaystyle P} such that it intersects the line g {\displaystyle g} in two points A , B {\displaystyle A,B} , and the perpendicular to be constructed is the one bisecting segment A B {\displaystyle AB} . If a → , b → {\displaystyle {\vec {a}},{\vec {b}}} are the position vectors of two points A , B {\displaystyle A,B} , then its midpoint is M : m → = a → + b → 2 {\displaystyle M:{\vec {m}}={\tfrac {{\vec {a}}+{\vec {b}}}{2}}} and vector a → − b → {\displaystyle {\vec {a}}-{\vec {b}}} is a normal vector of the perpendicular line segment bisector. Hence its vector equation is (x → − m →) · (a → − b →) = 0 {\displaystyle ({\vec {x}}-{\vec {m}})\cdot ({\vec {a}}-{\vec {b}})=0} . Inserting m → = ⋯ {\displaystyle {\vec {m}}=\cdots } and expanding the equation leads to the vector equation (V) x → · (a → − b →) = 1 2 (a → 2 − b → 2) . {\displaystyle \quad {\vec {x}}\cdot ({\vec {a}}-{\vec {b}})={\tfrac {1}{2}}({\vec {a}}^{2}-{\vec {b}}^{2}).} With A = (a 1 , a 2) , B = (b 1 , b 2) {\displaystyle A=(a_{1},a_{2}),B=(b_{1},b_{2})} one gets the equation in coordinate form: (C) (a 1 − b 1) x + (a 2 − b 2) y = 1 2 (a 1 2 − b 1 2 + a 2 2 − b 2 2) . {\displaystyle \quad (a_{1}-b_{1})x+(a_{2}-b_{2})y={\tfrac {1}{2}}(a_{1}^{2}-b_{1}^{2}+a_{2}^{2}-b_{2}^{2})\;.} Or explicitly: (E) y = m (x − x 0) + y 0 {\displaystyle \quad y=m(x-x_{0})+y_{0}} , where m = − b 1 − a 1 b 2 − a 2 {\displaystyle \;m=-{\tfrac {b_{1}-a_{1}}{b_{2}-a_{2}}}} , x 0 = 1 2 (a 1 + b 1) {\displaystyle \;x_{0}={\tfrac {1}{2}}(a_{1}+b_{1})\;} , and y 0 = 1 2 (a 2 + b 2) {\displaystyle \;y_{0}={\tfrac {1}{2}}(a_{2}+b_{2})\;} . Perpendicular line segment bisectors were used solving various geometric problems: Construction of the center of a Thales' circle, Construction of the center of the Excircle of a triangle, Voronoi diagram boundaries consist of segments of such lines or planes. Bisector plane The perpendicular bisector of a line segment is a plane, which meets the segment at its midpoint perpendicularly. Its vector equation is literally the same as in the plane case: (V) x → · (a → − b →) = 1 2 (a → 2 − b → 2) . {\displaystyle \quad {\vec {x}}\cdot ({\vec {a}}-{\vec {b}})={\tfrac {1}{2}}({\vec {a}}^{2}-{\vec {b}}^{2}).} With A = (a 1 , a 2 , a 3) , B = (b 1 , b 2 , b 3) {\displaystyle A=(a_{1},a_{2},a_{3}),B=(b_{1},b_{2},b_{3})} one gets the equation in coordinate form: (C3) (a 1 − b 1) x + (a 2 − b 2) y + (a 3 − b 3) z = 1 2 (a 1 2 − b 1 2 + a 2 2 − b 2 2 + a 3 2 − b 3 2) . {\displaystyle \quad (a_{1}-b_{1})x+(a_{2}-b_{2})y+(a_{3}-b_{3})z={\tfrac {1}{2}}(a_{1}^{2}-b_{1}^{2}+a_{2}^{2}-b_{2}^{2}+a_{3}^{2}-b_{3}^{2})\;.} Property (D) (see above) is literally true in space, too: (D) The perpendicular bisector plane of a segment A B {\displaystyle AB} has for any point X {\displaystyle X} the property: | X A | = | X B | {\displaystyle \;|XA|=|XB|} . Bisection of an angle using a compass and straightedge An angle bisector divides the angle into two angles with equal measures. An angle only has one bisector. Each point of an angle bisector is equidistant from the sides of the angle. The 'interior' or 'internal bisector' of an angle is the line, half-line, or line segment that divides an angle of less than 180° into two equal angles. The 'exterior' or 'external bisector' is the line that divides the supplementary angle (of 180° minus the original angle), formed by one side forming the original angle and the extension of the other side, into two equal angles.[1] To bisect an angle with straightedge and compass, one draws a circle whose center is the vertex. The circle meets the angle at two points: one on each leg. Using each of these points as a center, draw two circles of the same size. The intersection of the circles (two points) determines a line that is the angle bisector. The proof of the correctness of this construction is fairly intuitive, relying on the symmetry of the problem. The trisection of an angle (dividing it into three equal parts) cannot be achieved with the compass and ruler alone (this was first proved by Pierre Wantzel). The internal and external bisectors of an angle are perpendicular. If the angle is formed by the two lines given algebraically as l 1 x + m 1 y + n 1 = 0 {\displaystyle l_{1}x+m_{1}y+n_{1}=0} and l 2 x + m 2 y + n 2 = 0 , {\displaystyle l_{2}x+m_{2}y+n_{2}=0,} then the internal and external bisectors are given by the two equations[2]:p.15 l 1 x + m 1 y + n 1 l 1 2 + m 1 2 = ± l 2 x + m 2 y + n 2 l 2 2 + m 2 2 . {\displaystyle {\frac {l_{1}x+m_{1}y+n_{1}}{\sqrt {l_{1}^{2}+m_{1}^{2}}}}=\pm {\frac {l_{2}x+m_{2}y+n_{2}}{\sqrt {l_{2}^{2}+m_{2}^{2}}}}.} The interior angle bisectors of a triangle are concurrent in a point called the incenter of the triangle, as seen in the diagram. The bisectors of two exterior angles and the bisector of the other interior angle are concurrent.[3]:p.149 Three intersection points, each of an external angle bisector with the opposite extended side, are collinear (fall on the same line as each other).[3]:p. 149 Three intersection points, two of them between an interior angle bisector and the opposite side, and the third between the other exterior angle bisector and the opposite side extended, are collinear.[3]:p. 149 Main article: Angle bisector theorem In this diagram, BD:DC = AB:AC. The angle bisector theorem is concerned with the relative lengths of the two segments that a triangle's side is divided into by a line that bisects the opposite angle. It equates their relative lengths to the relative lengths of the other two sides of the triangle. If the side lengths of a triangle are a , b , c {\displaystyle a,b,c} , the semiperimeter s = (a + b + c) / 2 , {\displaystyle s=(a+b+c)/2,} and A is the angle opposite side a {\displaystyle a} , then the length of the internal bisector of angle A is[3]:p. 70 2 b c s (s − a) b + c , {\displaystyle {\frac {2{\sqrt {bcs(s-a)}}}{b+c}},} or in trigonometric terms,[4] 2 b c b + c cos ⁡ A 2 . {\displaystyle {\frac {2bc}{b+c}}\cos {\frac {A}{2}}.} If the internal bisector of angle A in triangle ABC has length t a {\displaystyle t_{a}} and if this bisector divides the side opposite A into segments of lengths m and n, then[3]:p.70 t a 2 + m n = b c {\displaystyle t_{a}^{2}+mn=bc} where b and c are the side lengths opposite vertices B and C; and the side opposite A is divided in the proportion b:c. If the internal bisectors of angles A, B, and C have lengths t a , t b , {\displaystyle t_{a},t_{b},} and t c {\displaystyle t_{c}} , then[5] (b + c) 2 b c t a 2 + (c + a) 2 c a t b 2 + (a + b) 2 a b t c 2 = (a + b + c) 2 . {\displaystyle {\frac {(b+c)^{2}}{bc}}t_{a}^{2}+{\frac {(c+a)^{2}}{ca}}t_{b}^{2}+{\frac {(a+b)^{2}}{ab}}t_{c}^{2}=(a+b+c)^{2}.} No two non-congruent triangles share the same set of three internal angle bisector lengths.[6][7] There exist integer triangles with a rational angle bisector. The internal angle bisectors of a convex quadrilateral either form a cyclic quadrilateral (that is, the four intersection points of adjacent angle bisectors are concyclic),[8] or they are concurrent. In the latter case the quadrilateral is a tangential quadrilateral. Each diagonal of a rhombus bisects opposite angles. The excenter of an ex-tangential quadrilateral lies at the intersection of six angle bisectors. These are the internal angle bisectors at two opposite vertex angles, the external angle bisectors (supplementary angle bisectors) at the other two vertex angles, and the external angle bisectors at the angles formed where the extensions of opposite sides intersect. Main article: Parabola § Tangent bisection property BE bisects ∠FEC The tangent to a parabola at any point bisects the angle between the line joining the point to the focus and the line from the point and perpendicular to the directrix. Each of the three medians of a triangle is a line segment going through one vertex and the midpoint of the opposite side, so it bisects that side (though not in general perpendicularly). The three medians intersect each other at a point which is called the centroid of the triangle, which is its center of mass if it has uniform density; thus any line through a triangle's centroid and one of its vertices bisects the opposite side. The centroid is twice as close to the midpoint of any one side as it is to the opposite vertex. Main article: Circumcircle The interior perpendicular bisector of a side of a triangle is the segment, falling entirely on and inside the triangle, of the line that perpendicularly bisects that side. The three perpendicular bisectors of a triangle's three sides intersect at the circumcenter (the center of the circle through the three vertices). Thus any line through a triangle's circumcenter and perpendicular to a side bisects that side. In an acute triangle the circumcenter divides the interior perpendicular bisectors of the two shortest sides in equal proportions. In an obtuse triangle the two shortest sides' perpendicular bisectors (extended beyond their opposite triangle sides to the circumcenter) are divided by their respective intersecting triangle sides in equal proportions.[9]:Corollaries 5 and 6 For any triangle the interior perpendicular bisectors are given by p a = 2 a T a 2 + b 2 − c 2 , {\displaystyle p_{a}={\tfrac {2aT}{a^{2}+b^{2}-c^{2}}},} p b = 2 b T a 2 + b 2 − c 2 , {\displaystyle p_{b}={\tfrac {2bT}{a^{2}+b^{2}-c^{2}}},} and p c = 2 c T a 2 − b 2 + c 2 , {\displaystyle p_{c}={\tfrac {2cT}{a^{2}-b^{2}+c^{2}}},} where the sides are a ≥ b ≥ c {\displaystyle a\geq b\geq c} and the area is T . {\displaystyle T.} [9]:Thm 2 The two bimedians of a convex quadrilateral are the line segments that connect the midpoints of opposite sides, hence each bisecting two sides. The two bimedians and the line segment joining the midpoints of the diagonals are concurrent at a point called the "vertex centroid" and are all bisected by this point.[10]:p.125 The four "maltitudes" of a convex quadrilateral are the perpendiculars to a side through the midpoint of the opposite side, hence bisecting the latter side. If the quadrilateral is cyclic (inscribed in a circle), these maltitudes are concurrent at (all meet at) a common point called the "anticenter". Brahmagupta's theorem states that if a cyclic quadrilateral is orthodiagonal (that is, has perpendicular diagonals), then the perpendicular to a side from the point of intersection of the diagonals always bisects the opposite side. The perpendicular bisector construction forms a quadrilateral from the perpendicular bisectors of the sides of another quadrilateral. There is an infinitude of lines that bisect the area of a triangle. Three of them are the medians of the triangle (which connect the sides' midpoints with the opposite vertices), and these are concurrent at the triangle's centroid; indeed, they are the only area bisectors that go through the centroid. Three other area bisectors are parallel to the triangle's sides; each of these intersects the other two sides so as to divide them into segments with the proportions 2 + 1 : 1 {\displaystyle {\sqrt {2}}+1:1} .[11] These six lines are concurrent three at a time: in addition to the three medians being concurrent, any one median is concurrent with two of the side-parallel area bisectors. The envelope of the infinitude of area bisectors is a deltoid (broadly defined as a figure with three vertices connected by curves that are concave to the exterior of the deltoid, making the interior points a non-convex set).[11] The vertices of the deltoid are at the midpoints of the medians; all points inside the deltoid are on three different area bisectors, while all points outside it are on just one. [1] The sides of the deltoid are arcs of hyperbolas that are asymptotic to the extended sides of the triangle.[11] The ratio of the area of the envelope of area bisectors to the area of the triangle is invariant for all triangles, and equals 3 4 log e ⁡ (2) − 1 2 , {\displaystyle {\tfrac {3}{4}}\log _{e}(2)-{\tfrac {1}{2}},} i.e. 0.019860... or less than 2%. A cleaver of a triangle is a line segment that bisects the perimeter of the triangle and has one endpoint at the midpoint of one of the three sides. The three cleavers concur at (all pass through) the center of the Spieker circle, which is the incircle of the medial triangle. The cleavers are parallel to the angle bisectors. A splitter of a triangle is a line segment having one endpoint at one of the three vertices of the triangle and bisecting the perimeter. The three splitters concur at the Nagel point of the triangle. Any line through a triangle that splits both the triangle's area and its perimeter in half goes through the triangle's incenter (the center of its incircle). There are either one, two, or three of these for any given triangle. A line through the incenter bisects one of the area or perimeter if and only if it also bisects the other.[12] Any line through the midpoint of a parallelogram bisects the area[11] and the perimeter. All area bisectors and perimeter bisectors of a circle or other ellipse go through the center, and any chords through the center bisect the area and perimeter. In the case of a circle they are the diameters of the circle. The diagonals of a parallelogram bisect each other. If a line segment connecting the diagonals of a quadrilateral bisects both diagonals, then this line segment (the Newton Line) is itself bisected by the vertex centroid. A plane that divides two opposite edges of a tetrahedron in a given ratio also divides the volume of the tetrahedron in the same ratio. Thus any plane containing a bimedian (connector of opposite edges' midpoints) of a tetrahedron bisects the volume of the tetrahedron[13][14]:pp.89–90
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The Angle Bisector at cut-the-knot Angle Bisector definition. Math Open Reference With interactive applet Line Bisector definition. Math Open Reference With interactive applet Perpendicular Line Bisector. With interactive applet Animated instructions for bisecting an angle and bisecting a line Using a compass and straightedge Weisstein, Eric W. "Line Bisector". MathWorld. This article incorporates material from Angle bisector on PlanetMath, which is licensed under the Creative Commons Attribution/Share-Alike License. Retrieved from " 1 Measure the angle. Place the origin hole of the compass on the angle’s vertex, lining up the base line with one of the angle’s rays. Look at the degree mark where the other ray falls. This will tell you the degree measurement of the angle.[1] For example, you might measure an angle that is 160 degrees wide. Remember that protractors have two sets of numbers. To know which set of numbers to look at, think about the size of the angle. Obtuse angles are greater than 90 degrees, and acute angles are less than 90 degrees. 2 Divide the number of degrees in half. An angle bisector divides an angle into two equal parts.[2] So, to find where the angle bisector lays, divide the number of degrees in the angle by 2.[3] For example, if the angle is 160 degrees, you would calculate 160 2 = 80 {\displaystyle {\frac {160}{2}}=80} . So, the angle bisector is at the 80-degree mark of the angle. Advertisement 3 Draw a point marking the measurement of the bisector. Again, line up the origin hole with the angle’s vertex, and line up the base line with one of the rays. Find the halfway point using the protractor. Mark this point in the angle’s interior.[4] For example, if the bisector of a 160-degree angle lies at the 80-degree point, you would find the 80-degree mark on the protractor and draw a point at this location in the angle’s interior. 4 Draw a line from the vertex to the point. Use the straightedge of the protractor to connect the vertex to the halfway point of the angle. The line you draw is the angle’s bisector.[5] Advertisement 1 Draw an arc across both rays. Open the compass to any width, and place the point of the compass at the angle’s vertex. Swing the compass so that the pencil draws an arc that crosses both rays of the angle.[6] For example, you might have angle BAC. Place the compass tip on point A. Swing the compass so that it draws an arc intersecting ray AB at point D, and ray AC at point E. 2 Draw an interior arc. Move the compass so that the point sits on the location where the first arc intersects the first ray. Swing the compass, drawing an arc inside of the angle.[7] For example, place the compass tip on point D and draw an arc inside the angle. 3 Draw a second interior arc intersecting the first interior arc. Without changing the width of the compass, move the point to the location where the first arc intersects the second ray. Swing the compass, drawing an interior arc that intersects the first interior arc you drew.[8] For example, place the compass tip on point E and draw an arc intersecting the first interior arc. Label the point of their intersection point F. 4 Draw a line from the vertex to the point where the arcs intersect. Use a straightedge to ensure that the line is accurate. This line bisects the angle.[9] For example, use a straightedge to draw a line connecting points F and A. Advertisement Add New Question Question What is the number of bisectors that can be drawn of a given angle? One. Question How do I prove that the ray drawn bisects the angle? Check the angles between either of the original rays and the bisector line. If they are equal, then the line is the exact bisector. Question How can I draw more than one angle bisector? Any given angle has exactly one bisector. See more answers Ask a Question Advertisement This article was reviewed by Joseph Meyer. Joseph Meyer is a High School Math Teacher based in Pittsburgh, Pennsylvania. He is an educator at City Charter High School, where he has been teaching for over 7 years. Joseph is also the founder of Sandbox Math, an online learning community dedicated to helping students succeed in Algebra. His site is set apart by its focus on fostering genuine comprehension through step-by-step understanding (instead of just getting the correct final answer), enabling learners to identify and overcome misunderstandings and confidently take on any test they face. He received his MA in Physics from Case Western Reserve University and his BA in Physics from Baldwin Wallace University. This article has been viewed 245,645 times. Co-authors: 10 Updated: December 16, 2022 Views: 245,645 Categories: Geometry Print Send fan mail to authors Thanks to all authors for creating a page that has been read 245,645 times. "I always struggle with constructing a bisector with both compass and protractor, but wikiHow made it so much easier with only 4 steps. I was actually scrolling down thinking there was more!"..." more Share your story A line that splits an angle into two equal angles. ("Bisect" means to divide into two equal parts.) Try moving the points below, the red line is the Angle Bisector: An angle bisector is defined as a ray, segment, or line that divides a given angle into two angles of equal measures. The word bisector or bisection means dividing one thing into two equal parts. In geometry, we divide an angle by a line or ray which is considered as an angle bisector. What is Angle Bisector? The angle bisector in geometry is the ray, line, or segment which divides a given angle into two equal parts. For example, an angle bisector of a 60-degree angle will divide it into two angles of 30 degrees each. In other words, it divides an angle into two smaller congruent angles. Given below is an image of an angle bisector of ∠AOB. Angle Bisector of a Triangle In a triangle, the angle bisector of an angle is a straight line that divides the angle into two equal or congruent angles. There can be three angle bisectors in every triangle, one for each vertex. The point where these three angle bisectors meet in a triangle is known as its incenter. The distance between the incenter to all the edges of a triangle is the same. Look at the image below showing the angle bisector of a triangle. Here, AG, CE, and BD are the angle bisectors of ∠BAC, ∠ACB, and ∠ABC respectively. F is the point of intersection of all three bisectors which is known as incenter and it is at an equal distance from each of the vertex. Properties of an Angle Bisector Till now you must be clear about the meaning of angle bisector in geometry. Now, let us learn some of the angle bisector properties listed below: An angle bisector divides an angle into two equal parts. Any point on the bisector of an angle is equidistant from the sides or arms of the angle. In a triangle, it divides the opposite side into the ratio of the measure of the other two sides. Construction of Angle Bisector Let's try constructing the angle bisector for an angle. In this section, we will see the steps to be followed for angle bisector construction. Steps to Construct an Angle Bisector: Step 1: Draw any angle, say ∠ABC. Step 2: Taking B as the center and any appropriate radius, draw an arc to intersect the rays BA and BC at, say, E and D respectively. (Refer to the figure below) Step 3: Now, taking D and E as centers and with the same radius as taken in the previous step, draw two arcs to intersect each other at F. Step 4: Join B to F and extend it as a ray. This ray BF is the required angle bisector of angle ABC. Angle Bisector Theorem Let's now understand in detail an important property of the angle bisector of a triangle as stated in the previous section. This property is known as the angle bisector theorem of a triangle. According to the angle bisector theorem, in a triangle, the angle bisector drawn from one vertex divides the side on which it falls in the same ratio as the ratio of the other two sides of the triangle. Statement: An angle bisector of a triangle divides the opposite side into two segments that are proportional to the other two sides of the triangle. In the above image, PS is the angle bisector of ∠P in ΔPQR. Therefore, by applying the angle bisector theorem we can say that PQ/PR = QS/SR or a/b = x/y. ► Related Articles Check these interesting articles related to the angle bisector in math. Example 1: In the figure given below, BD is the bisector of ∠ABC and BE bisects ∠ABD. Find the measure of ∠DBE given that ∠ABC=80°. Solution: It is given that ∠ABC=80°. Also, ∠ABD = 1/2 × ∠ABC = 1/2 × 80° = 40° (BD is an angle bisector bisecting ∠ABC into two equal parts) Now, ∠DBE = 1/2 × ∠ABD = 1/2 × 40° = 20° (BE is a bisector and bisects ∠ABD into two equal parts) ∴ The value of ∠DBE is 20°. Example 2: In the figure, the ray drawn from point O is the angle bisector of ∠BON. Find x. Solution: To find x, we will be using the property: Any point on the bisector of an angle is equidistant from the sides of the angle. So, the bisector drawn from O will be equidistant from sides OB and ON. ⇒ 3x − 2 = 10 ⇒ 3x = 2 + 10 ⇒ 3x = 12 ∴ The value of x is 4. Example 3: If QS is the bisector of ∠PQR, find x. Solution: As QS bisects ∠PQR, by angle bisector theorem we get, QP/QR = PS/RS. ⇒ 18/24 = 12/x ⇒ x = (12 × 24)/18 ⇒ x = (2 × 24)/3 = 2 × 8 = 16 ∴ The value of x is 16. Show Solution > go to slidego to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations with Cuemath. Book a Free Trial Class FAQs on Angle Bisector An angle bisector is the ray, line, or line segment which divides an angle into two congruent angles. What are the Properties of Angle Bisector? An angle bisector has two main properties: Any point on the bisector of an angle is equidistant from the sides of the angle. In a triangle, the angle bisector divides the opposite side in the ratio of the adjacent sides. What is an Angle Bisector of a Triangle? The angle bisector of a triangle drawn from any of the three vertices divides the opposite side in the ratio of the other two sides of the triangle. There can be three angle bisectors drawn in a triangle. Does Angle Bisector Cut an Angle in Half? Yes, an angle bisector divides the given angle into two equal angles. In other words, we can say that the measure of each of these angles is half of the original angle. How to Construct an Angle Bisector? An angle bisector construction can be done by following the steps given below: Step 1: Take a compass and take any suitable width on it. Place its tip on the vertex of the angle and draw an arc touching the arms of the angle at two distinct points. Step 2: Keep the same width of the compass and draw arcs intersecting each other from each of those two points. Step 3: Draw a ray from the vertex of the angle to the point of intersection formed in the previous step. Step 4: That ray will be the required angle bisector of the given angle. What is the Property of Angle Bisector of Triangle? The property of the angle bisector of a triangle states that the angle bisector divides the opposite side of a triangle in the ratio of its adjacent sides. Does the Angle Bisector go through the Midpoint? It is not always true that an angle bisector goes through the midpoint of the opposite side. It divides the opposite side in proportion to the adjacent sides of the triangle. Can an Angle have More Than One Angle Bisector? No, an angle can have only one angle bisector. For example, if we bisect a 60° angle we will get two 30° angles as a result. This means 60° angle is divided into two equal angles (30° each). Hence, 60° angle can only be bisected once. Further, we can again bisect 30° angle into two equal angles as 15° each. An angle bisector is defined as a ray, segment, or line that divides a given angle into two angles of equal measures. The word bisector or bisection means dividing one thing into two equal parts. In geometry, we divide an angle by a line or ray which is considered as an angle bisector. What is Angle Bisector? The angle bisector in geometry is the ray, line, or segment which divides a given angle into two equal parts. For example, an angle bisector of a 60-degree angle will divide it into two angles of 30 degrees each. In other words, it divides an angle into two smaller congruent angles. Given below is an image of an angle bisector of ∠AOB. Angle Bisector of a Triangle In a triangle, the angle bisector of an angle is a straight line that divides the angle into two equal or congruent angles. There can be three angle bisectors in every triangle, one for each vertex. The point where these three angle bisectors meet in a triangle is known as its incenter. The distance between the incenter to all the edges of a triangle is the same. Look at the image below showing the angle bisector of a triangle. Here, AG, CE, and BD are the angle bisectors of ∠BAC, ∠ACB, and ∠ABC respectively. F is the point of intersection of all three bisectors which is known as incenter and it is at an equal distance from each of the vertex. Properties of an Angle Bisector Till now you must be clear about the meaning of angle bisector in geometry. Now, let us learn some of the angle bisector properties listed below: An angle bisector divides an angle into two equal parts. Any point on the bisector of an angle is equidistant from the sides or arms of the angle. In a triangle, it divides the opposite side into the ratio of the measure of the other two sides. Construction of Angle Bisector Let's try constructing the angle bisector for an angle. In this section, we will see the steps to be followed for angle bisector construction. Steps to Construct an Angle Bisector: Step 1: Draw any angle, say ∠ABC. Step 2: Taking B as the center and any appropriate radius, draw an arc to intersect the rays BA and BC at, say, E and D respectively. (Refer to the figure below) Step 3: Now, taking D and E as centers and with the same radius as taken in the previous step, draw two arcs to intersect each other at F. Step 4: Join B to F and extend it as a ray. This ray BF is the required angle bisector of angle ABC. Angle Bisector Theorem Let's now understand in detail an important property of the angle bisector of a triangle as stated in the previous section. This property is known as the angle bisector theorem of a triangle. According to the angle bisector theorem, in a triangle, the angle bisector drawn from one vertex divides the side on which it falls in the same ratio as the ratio of the other two sides of the triangle. Statement: An angle bisector of a triangle divides the opposite side into two segments that are proportional to the other two sides of the triangle. In the above image, PS is the angle bisector of ∠P in ΔPQR. Therefore, by applying the angle bisector theorem we can say that PQ/PR = QS/SR or a/b = x/y. ► Related Articles Check these interesting articles related to the angle bisector in math. Example 1: In the figure given below, BD is the bisector of ∠ABC and BE bisects ∠ABD. Find the measure of ∠DBE given that ∠ABC=80°. Solution: It is given that ∠ABC=80°. Also, ∠ABD = 1/2 × ∠ABC = 1/2 × 80° = 40° (BD is an angle bisector bisecting ∠ABC into two equal parts) Now, ∠DBE = 1/2 × ∠ABD = 1/2 × 40° = 20° (BE is a bisector and bisects ∠ABD into two equal parts) ∴ The value of ∠DBE is 20°. Example 2: In the figure, the ray drawn from point O is the angle bisector of ∠BON. Find x. Solution: To find x, we will be using the property: Any point on the bisector of an angle is equidistant from the sides of the angle. So, the bisector drawn from O will be equidistant from sides OB and ON. ⇒ 3x − 2 = 10 ⇒ 3x = 2 + 10 ⇒ 3x = 12 ∴ The value of x is 4. Example 3: If QS is the bisector of ∠PQR, find x. Solution: As QS bisects ∠PQR, by angle bisector theorem we get, QP/QR = PS/RS. ⇒ 18/24 = 12/x ⇒ x = (12 × 24)/18 ⇒ x = (2 × 24)/3 = 2 × 8 = 16 ∴ The value of x is 16. Show Solution > go to slidego to slidego to slide Breakdown tough concepts through simple visuals. Math will no longer be a tough subject, especially when you understand the concepts through visualizations with Cuemath. Book a Free Trial Class FAQs on Angle Bisector An angle bisector is the ray, line, or line segment which divides an angle into two congruent angles. What are the Properties of Angle Bisector? An angle bisector has two main properties: Any point on the bisector of an angle is equidistant from the sides of the angle. In a triangle, the angle bisector divides the opposite side in the ratio of the adjacent sides. What is an Angle Bisector of a Triangle? The angle bisector of a triangle drawn from any of the three vertices divides the opposite side in the ratio of the other two sides of the triangle. There can be three angle bisectors drawn in a triangle. Does Angle Bisector Cut an Angle in Half? Yes, an angle bisector divides the given angle into two equal angles. In other words, we can say that the measure of each of these angles is half of the original angle. How to Construct an Angle Bisector? An angle bisector construction can be done by following the steps given below: Step 1: Take a compass and take any suitable width on it. Place its tip on the vertex of the angle and draw an arc touching the arms of the angle at two distinct points. Step 2: Keep the same width of the compass and draw arcs intersecting each other from each of those two points. Step 3: Draw a ray from the vertex of the angle to the point of intersection formed in the previous step. Step 4: That ray will be the required angle bisector of the given angle. What is the Property of Angle Bisector of Triangle? The property of the angle bisector of a triangle states that the angle bisector divides the opposite side of a triangle in the ratio of its adjacent sides. Does the Angle Bisector go through the Midpoint? It is not always true that an angle bisector goes through the midpoint of the opposite side. It divides the opposite side in proportion to the adjacent sides of the triangle. Can an Angle have More Than One Angle Bisector? No, an angle can have only one angle bisector. For example, if we bisect a 60° angle we will get two 30° angles as a result. This means 60° angle is divided into two equal angles (30° each). Hence, 60° angle can only be bisected once. Further, we can again bisect 30° angle into two equal angles as 15° each. Join the vertex with the point where the arcs intersect. Using a straight-edge – a ruler, join up the point where the arcs intersect each other with the vertex Q . The new straight line is the angle bisector of the original angle PQR . You can check that the new straight line bisects the angle PQR by using a protractor to measure. The angle ABC should have been cut into two equal angles. The two new angles are congruent.